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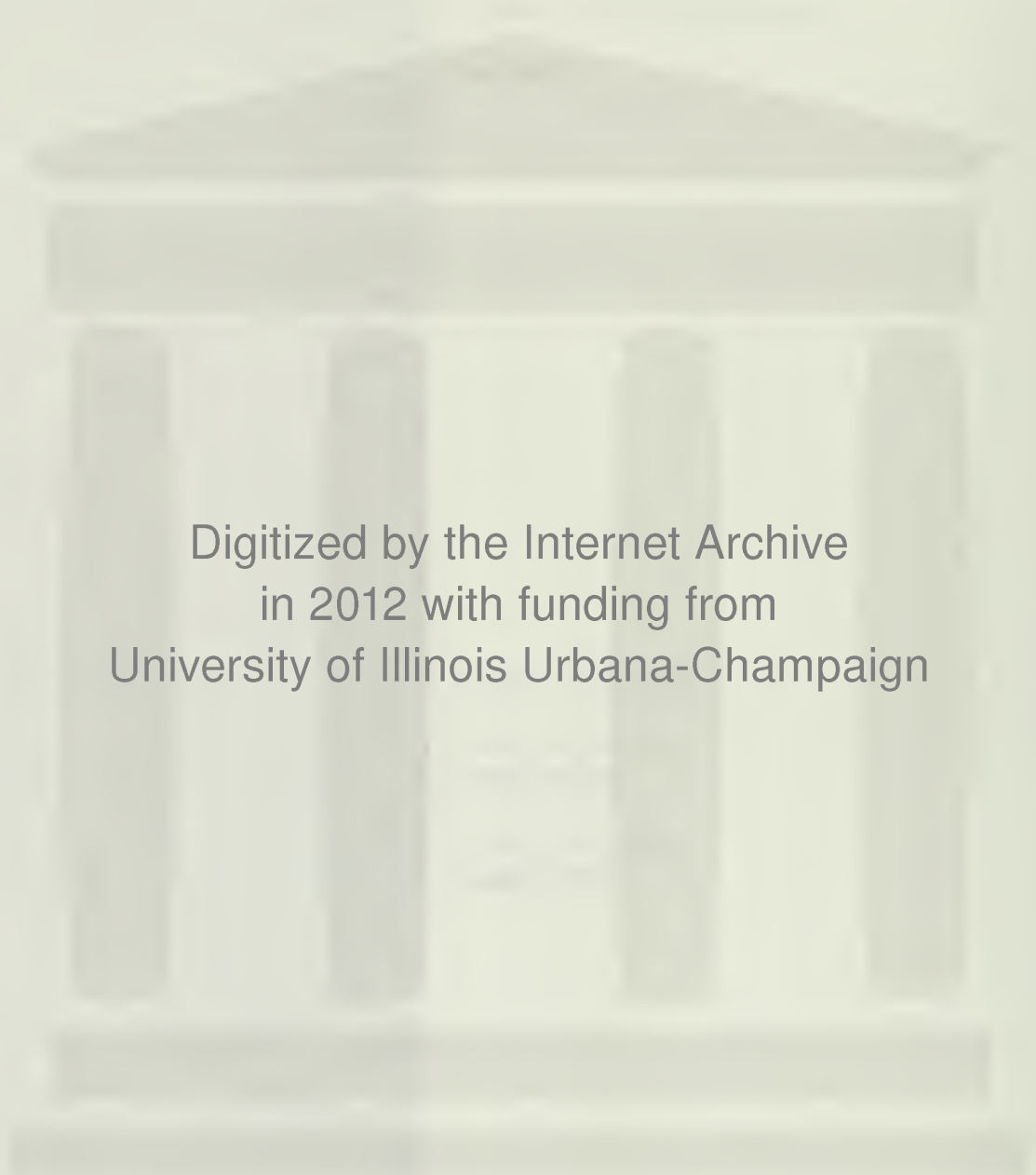
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An Integrated Cash Flow Model of the Firm

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ABSTRACT

This paper develops an integrated model that links short-run financial management (SRFM) information to the net present value approach. The model is developed within the framework of value maximization under conditions of certainty and uncertainty. The objectives are to expand the model of Sartoris and Hill by including all of the key SRFM variables, developing interrelationships among various SRFM variables and introducing the effect of forecasting errors on inventories, net cash flow shortfalls and excesses. A timeline is developed to highlight the effect of a firm's competitive position on its credit terms and the speed that it collects and disburses cash. A primary contribution of the paper is to show that SRFM variables affect the magnitude and timing of cash flows and are directly related to the value creation process. By integrating the SRFM variables into the long-run financial planning process, fresh insights concerning the creation of firms' value are introduced. In summary, the model highlights the complex interdependencies that exist between SRFM variables and firm value.

AN INTEGRATED CASH FLOW MODEL OF THE FIRM

I. INTRODUCTION

The focus of corporate finance literature is on long-run financial management issues such as capital structure, cost of capital, dividend policy and valuation of the firm. Until recently the short-run financial management (SRFM) literature was not integrated into corporate finance theory. Traditionally, the SRFM literature has been devoted to issues that often have a single focus, e.g., the theoretical literature related to cash management developed by Baumol [1], Miller and Orr [23] or Stone [31], or recent cash management literature such as Ferguson and Mairer [8], Stone [34], or Stone and Hill [35]. The credit/receivables management literature has a relatively narrow perspective, e.g., there is an extensive literature developed on the management of receivables that features Benishay [4], Carpenter and Miller [5], Freitas [6], Gentry and De La Garza [11], Greer [12], Halloran and Lanser [13], Hill and Reiner [14], Kallberg and Saunders [15], Kim and Atkins [16], Lewellen and Edmister [18], Lewellen and Johnson [19], Lewellen, McConnell and Scott [20], Lieber and Orgler [21], Mehta [22], Sachdeva and Gitman [25], Sartoris and Hill [26], Stone [32], and Weston and Tuan [36].

The literature has segmented the major SRFM activities into functional areas such as the management of cash, receivables/credit, inventories/purchases or short-run financial forecasting which focuses on borrowing and investing decisions. Why did this segmentation occur and is it changing? Operationally these SRFM activities may

be separate because organizational structure prevents interaction among departments. However, the development of integrated computer information systems may be breaking down these previously existing organizational barriers. Additionally, the accounting segmentation of the balance sheet into current and noncurrent information is partially responsible for the lack of integration of SRFM flows into financial valuation models. Specialists in cash management have shown that cash inflow and cash outflow information is as important for decision making and tracking performance as is the use of accrual accounting information.

Recently a few authors have developed theoretical linkages between two or more SRFM activities. For example, Schiff and Lieber [28] used a value maximization framework to develop an integrated dynamic model for accounts receivable and inventory management. However, they did not include the concept of the time value of money. Other examples are in Stone's models [30, 33] that link the cash budget, credit requirement determination and bank system design in determining the optimal banking system for the firm; Knight [17] limits the role of optimization among current asset accounts by focusing on the uncertainties that exist in the simultaneous interrelationship among investment, profit and risk; the linkage between cash, credit management and short term financing is well developed by Stone [29, 30, 33]. Emery [6] and Emery and Cogger [7] focus on linking the measurement of liquidity to the behavior of cash flow information, while Richards and Laughlin [24] relate liquidity to the cash conversion cycle. Finally, Bierman, Chopra and Thomas [3] link optimal

working capital to capital structure through a risk of ruin model. Models designed to determine a partial equilibrium among a small set of variables are frequently criticized because finding the optimal solution for a set of variables may result in a nonoptimal solution for another set of relationships. In the current state of knowledge, an optimal solution for the total SRFM process has not been determined.

In 1980, Gentry [10] used a simulation approach in a value maximization framework to integrate all working capital components into the capital budgeting process. The model highlighted inflationary effects, forecasting errors and uncertainty. Using a numerical example, the simulation model demonstrated that the exclusion of working capital components from the capital investment analysis could result in an overstatement of a project's discounted value and may lead to errors in the accept/reject decision of a capital investment.

Recently, Sartoris and Hill (S&H) [27] integrated short-run cash inflows and outflows into the net present value model. A significant contribution of the model was to show that changes in short-run financial management policies had a direct effect on the value of the firm. The S&H model made a significant contribution to the corporate finance literature because it established a theoretical linkage between short- and long-run financial management. The S&H model sets the stage for building new theoretical linkages between short- and long-run financial management and it stimulates a natural evolution of thought on the subject of short-run financial management.

The primary objective of this paper is to develop an integrated model that links short-run financial management information to the NPV approach within the framework of value maximization under conditions of certainty and uncertainty. The plan is to expand the S&H model to include the important short-run financial variables and other concepts that have a significant effect on the value of the firm. The model provides an overview of the numerous cash inflow and outflow accounts and shows how they create or destroy value. The expanded model takes into account the interrelationships among the various short-run financial variables; collection and sales patterns, plus joint effects related to cash inflows; disbursement and purchase patterns plus joint effects related to cash outflows; and the forecasting error effect and its impact on inventories and cash flow shortfall (borrowing) or excess cash flow (lending). The model shows that without considering the above effects the theoretical value of the firm could be overstated. Unknowningly the firm could change its SRFM policies and mistakenly lower its value. The mistake is most serious with the existence of forecasting errors.

II. THE MODEL

A. Assumptions and Framework

The assumptions used in the development of the model are:

1. The firm is operating in a competitive factor market setting so that its short-term financial policies cannot affect raw material costs, inventory holding costs and other variable costs. However, these prices can vary over the planning horizon as a result of forces

external to the firm such as inflation and changes in the income level of the economy.

2. The firm is competing in a semi-competitive product market setting, such as monopolistic competitive market structure, which limits its ability to determine the price of its product. However, the firm can affect the effective price by changing its credit policy variables.

3. Credit terms to customers or from suppliers such as trade discounts, discount periods and credit periods, once determined by the firm, remain unchanged throughout the planning horizon.

4. For a specified planning horizon and product line firms adopt identical production technologies that result in production levels falling within a constant return-to-scale range for different credit policy alternatives.

5. Ordering costs for raw materials are negligible relative to the holding costs of inventory.

6. Rates of interest and return, along with the production period, the wage deferral period and the planning horizon are assumed to be exogenously determined.

7. There are no taxes.

8. The model assumes continuous compounding. Depending on the preference of the model user, the unit time period can be a day, week, month, quarter or year.

The preceding assumptions are designed to be realistic and to expand the valuation model to include all cash inflows and outflows related to a firm's real asset management in a competitive environment with a special emphasis on SRFM. The assumptions make it possible to

focus on and determine if the level and speed of the flows are creating or destroying value, and to evaluate the effect that demand and both short and long run financial policies have on the value of the firm. Finally, the assumptions provide a framework to measure the change in firm value that is related to sales forecasting errors and the resulting change in inventories, receivables and payables.

Since the sales level plays the linking role in the interrelationships among the SRFM variables in the model, it is important to understand its implicit functional form and its relationships with other variables before we move into the development and analysis of the model. According to the theory of demand, the sales (demand) level of the product is a function of its effective price level, the credit policy of the firm and its price level, i.e., $Q(t) = f(P(t), d, t_d, t_c)$ where $\frac{\partial Q(t)}{\partial P(t)} < 0$ and $\frac{\partial Q(t)}{\partial d}, \frac{\partial Q(t)}{\partial t_d}, \frac{\partial Q(t)}{\partial t_c} > 0$. In other words, the sales level is negatively associated with the effective price level of the product. Implicitly, we assume that other variables affecting the demand level are held constant, i.e., the aggregate income level of the economy, the taste of the customers as well as the prices of complementary and substitute goods.

B. Overview of the Model

The proposed model is an expansion of the theoretical valuation model developed by John Burr Williams [37] that utilizes the net present value (NPV) technique. The model is designed to show that changes in SRFM policies, product demand and costs of production can either create or destroy value of the firm. To create value the objective is

to collect inflows as quickly as possible and to hold onto the funds as long as possible, therefore, in the model cash inflows and outflows are separate inputs. Policies or actions that increase the speed (timing) of the inflows or increase the level (amount) of the inflows enhance the value of the firm and vice versa. Following is a listing of possible reasons that would cause an increase or a decrease in cash inflows and/or cash outflows.

Cash inflows are usually increased by (1) higher sales resulting from either an increase in price and/or quantity of goods sold, (2) an aggressive collection program designed to speed up the inflows, (3) a higher discount rate combined with a shorter discount period, (4) a reduction in bad debt and (5) higher returns on marketable securities. Cash inflows are also increased by the (6) sale of fixed assets or (7) the sale of common stock or debt.

Cash outflows are directly affected by the costs associated with (1) raw material, (2) goods-in-process, (3) finished goods, (4) capital expenditures, (5) variable costs, (6) interest on short term debt, (7) compensating balances and (8) disbursement terms to suppliers.

Usually, operating flows compose the major cash inflows or outflows to a firm. A timeline is developed in Exhibit 1 to show the inflows and outflows that exist in the cash operating cycle. The exhibit highlights how a firm's competitive position in its product market affects the speed of the flow of cash into and out of the firm, which directly affects the value of the firm. It is assumed in Exhibit 1 that a firm has one division and one set of credit terms for its customers and one set for its suppliers. Also, it is assumed the terms remain constant

throughout the planning period. Three scenarios are presented to illustrate how competition affects the timing of cash inflows and outflows.

The production phase of the timeline is presented at the top of Exhibit 1. It is assumed the production cycle is identical for all three scenarios. The assumption is that the receipt of an order triggers the production process and simultaneously materials and supplies are ordered on day 1. The raw materials (RM) are delivered at the end of day 13 and production does not start until the beginning of day 20. The raw material cycle is six days. The goods-in-process (GIP) cycle runs from day 14 through day 23 which is a 10 day cycle. The finished goods (FG) cycle is four days, day 30 through day 33 and encompasses the storage and delivery activities. The production/inventory cycle is the sum of the RM, GIP and FG cycle, which is 20 days. Although it is not shown in Exhibit 1, value has been added throughout the inventory cycle with outflows for labor and other production costs occurring throughout the GIP cycle. Additionally, there are other cash outflows not shown in Exhibit 1 that effect the value of the firm because of independent billing schedules, e.g., electric power, gas, communication, insurance and capital investment expenditures.

Scenario 1 depicts a firm that has received terms from its suppliers of 1/10, net 30. Payment to the supplier is deferred (P.D. period) until day 24 if the discount is taken or until day 44 if it is not taken. Cash is received from the customer on day 44 if the discount of 2/10, net 30 is taken and on day 64 if the discount is not taken. If a customer does not pay in 90 days the account is assumed to

EXHIBIT 1

CASH FLOW TIMELINE

Customer Order Received
Order Sent to Suppliers

Labor and Other
Production Costs

Delivery and
Storage Charges

1 14 20 24 30 34 44 64 94 124

Timeline - Day

Production Cycle

RM
RM Delivered

GIP

FG
Prod. Ends

Scenario 1

Cash Inflow

Cash Received (Discount)

Cash Received (No Discount)

AR (No Discount)

Bad Debt

Cash Outflow

P. D. Period (Discount)

AR (Discount)

P. D. Period (No Discount)

Supplier Paid

Scenario 2

Cash Inflow

Cash Received (Discount)

Cash Received (No Discount)

AR (No Discount)

Bad Debt

Cash Outflow

Supplier Paid

AR (Discount)

Scenario 3

Cash Inflow

Cash Received

P. D. Period (Discount)

Cash Outflow

P. D. Period (No Discount)

Supplier Paid

be a bad debt and no cash inflow occurs. The cash operating cycle is 30 days if the customer takes the discount and 50 days if the discount is not taken and payment is made on day 64. To complete the cash conversion cycle (CCC) the payment deferral period is subtracted from the operating cycle. The credit terms available and the decision to accept or reject the discount have a significant effect on the timing of the cash flows and thus the value of a firm. For example, if the discount from the supplier is taken and the customer takes the discount, the CCC is 20 days (day 44 - day 24). However, if the customer does not take the discount the CCC is 40 days (day 64 - day 24). Alternatively, if the discount from the supplier is not taken and the customer takes the discount, the CCC is zero (day 44 - day 44), but if the customer does not take the discount the CCC is 20 days (day 64 - day 44).

Scenario 2 exemplifies the firm that is in a weak bargaining position with both its customers and suppliers. The company has to pay the supplier cash on delivery (COD) on day 14. There is no discount when the terms are COD. Terms to the customers are 2/60, net 90. Thus, if the discount is taken the customer pays on day 94 which is an 80 day operating cycle, day 94 - day 14. The CCC is also 80 days because there is no payment deferral period. If the customer does not take the discount the CCC is 110 days.

Scenario 3 is representative of a firm that is in a strong bargaining position with both its suppliers and customers. The customer delivers a check for good funds on day 1 when the order is placed. The firm has the use of these funds until the suppliers are paid. The supplier provides terms of 2/30, net 60. If the firm takes the discount, the

operating cycle and CCC are -43 days (day 1 - day 44), which means the firm has the use of the funds for 43 days before the supplier is paid. The CCC is -73 days if the discount from the supplier is not taken.

The three scenarios illustrate that the competitive position of the firm to its suppliers and customers has a significant effect on the timing of the cash flows which directly effects the value of the company. Although not demonstrated in Exhibit 1, if a firm is in a strong competitive position with its customers the quantity of goods sold should be higher than if it was not in a strong competitive position. Likewise, if a firm was in a strong competitive position with its suppliers it could have lower costs of production, thereby reducing the cash outflow. Although the timeline in Exhibit 1 does not include all of the outflows or other revenue inflows, they also directly impact the value of the firm.

C. Certainty Model

In a world of certainty, the firm knows the exact timing and magnitude of its cash flows. There is no risk in this model since the values of the parameters and the variables in the model are known, therefore, the discount factor is the risk-free rate.

For ease in understanding the development of the model, a detailed discussion of the model's individual cash flow components for each period are presented. The sequence of the presentation follows the working capital elements as listed in a typical balance sheet.

1. Cash and Marketable Securities

There are several reasons for a firm to keep a cash balance even in conditions of certainty. For instance, although there is no speculative nor precautionary demand for cash, the transaction demand for cash plays a role in the certainty world. In addition, although some corporations are paying for all or part of the services rendered by their banks, many companies use compensating balances to pay for some or all of their bank service fees. Banks impute an interest charge, known as an earnings credit rate, on the balance to compensate for services (Burr [4]). For simplicity, it is assumed that the firm will keep a target cash balance (CM), to meet the above demands for cash where $CM = f(Q)$ and $Q = \int_0^T Q(s) * ds$, namely, the target cash level is a function of the total output level in the planning horizon. In terms of flows, it can be expressed as $-CM$. The notation used to describe each variable in the model is found in Appendix I.

It is assumed that in each period the firm invests the net cash level in excess of the target cash balances in marketable securities which have a one-period rate of return, j , which produces a cash inflow of $j * \text{MAX}(0, CA(t) - CM)$ referred to as MSCI. The net cash level $CA(t)$ in each period is an accumulated account of all past net cash inflows. It represents the cash collected from sales minus all expenses in all previous periods, i.e., $CA(t) = \int_0^t [CIF(s) - COF(s)] * ds$, where $CIF(s)$ is cash inflow in period s and $COF(s)$ is cash outflow in period s .

2. Collection of Accounts Receivable

The level of accounts receivable depends on the price, the sales level and the credit terms offered to the customers, which are the

trade discount, the discount and credit periods, i.e., $(AR = f(P(t), Q(t), d, t_d, t_c))$. It is believed that, other things unchanged, the level of accounts receivable rises with increases in price, sales level, length of the credit period, and decline with the level of trade discount and the length of the discount period, i.e.,

$$\frac{\partial AR}{\partial d}, \frac{\partial AR}{\partial t_d} < 0 \text{ and } \frac{\partial AR}{\partial P(t)}, \frac{\partial AR}{\partial Q(t)}, \frac{\partial AR}{\partial t_c} > 0.$$

The collection of accounts receivable at different points in time generates the firm's major cash inflows. The cash flows associated with the collection of accounts receivable reflect the payment pattern proportions of credit sales that are received by the firm. It is assumed that the payment pattern of customers, $q(t)$ of sales, can be either an identical pattern for all customers or it can reflect a distribution of customer payment patterns information.¹ In this model, it is assumed that a portion of credit sales is collected at the end of the discount period $(P(t)*(1-d)*q(t)*e^{-rt_d})$, simplified to dCI, and the remainder are collected at the end of the credit period after being adjusted for the bad debt loss, $b(t)$, $(P(t)*(1-q(t))*(1-b(t))*e^{-rt_c})$; referred to as ACI. The cash inflows associated with the collection of accounts receivable are expressed as $(P(t)*(1-d)*q(t)*e^{-rt_d} + P(t)*(1-q(t))*(1-b(t))*e^{-rt_c}) * Q(t)$, or simplified to $((dCI) + (ACI)) * Q(t)$.²

3. Inventory

In a certain world, there should not be any finished goods inventory because the firm produces exactly the quantity it sells. However,

throughout the production process, raw material and goods-in-process inventories exist and give rise to inventory holding costs. It is assumed that these costs are paid when the product is shipped at the end of the production period. This cash outflow is expressed as $-H(t)*Q(t)*e^{-rt}$, simplified to $HCO*Q(t)$.

4. Payment of Accounts Payable

Accounts payable reflect purchases of raw materials from the suppliers on credit rather than in cash. The payment of accounts payable is a major cash outflow for the firm. The cash flows associated with accounts payable represent the portion of raw material costs paid by the firm at different dates. That is, $AP = f(C(t), Q(t), c, t_p, t_f)$. It is believed, ceteris paribus, that the level of accounts payable rises with increase on raw material costs, sales level, length of credit period, and declines with the level of discount and the length of the discount period, i.e.,

$$\frac{\partial AP}{\partial c}, \frac{\partial AP}{\partial t_p} < 0 \text{ and } \frac{\partial AP}{\partial C(t)}, \frac{\partial AP}{\partial Q(t)}, \frac{\partial AP}{\partial t_f} > 0.$$

Similarly, it is assumed that the disbursement pattern to suppliers for purchases, $p(t)$, can be either an identical pattern to all suppliers or it can be a distribution of disbursement patterns to suppliers. It is assumed that a portion of raw material costs is paid at the end of the discount period, $(C(t)*(1-c)*p(t)*e^{-rt_p})$ or (cCO) and the rest is paid at the end of the credit period offered by the suppliers, $(C(t)*(1-p(t))*e^{-rt_f})$ or (ACO) . It is assumed that an ongoing healthy firm pays within the credit period in order to maintain a positive

relationship with its suppliers. Hence, the outflows associated with account payables, which are a major cash outflow of the firm, can be expressed as

$$-(C(t)*(1-c)*p(t)*e^{-rt} + C(t)*(1-p(t))*e^{-rt_f})*Q(t),$$

or $-((cCO) + (ACO))*Q(t)).$

5. Fixed Costs (for both models)

In this model, it is assumed that the cash outflows associated with fixed expenditures are composed of two parts. The first part is a lump sum fixed cash expenditure, FC, for machinery and equipment at the very beginning of the life of the plan, which is not salvageable. The other part is a series of periodic fixed cash outlays, FC(τ), throughout the life of the planning period. They represent the fixed expenses associated with nonoperating and overhead costs of production. It is assumed that these fixed expenses are paid at the end of each of the m equal-length periods, where $m = \frac{\text{project life (T)}}{\text{payment period } (\theta)}$.

In other words, the cash outflow components associated with fixed expenditures can be expressed as $-FC - FC(\tau)*e^{-rt}$ where

$$\begin{cases} FC(\tau) = FC(\tau) \text{ if } \tau = \frac{mt}{T} = 1, 2, \dots, m \\ FC(\tau) = 0 \text{ otherwise.} \end{cases}$$

6. Other Cash Outflows

It is assumed that the other variable costs, mainly labor costs, $W(t)*Q(t)$, are paid at the end of the deferral period. This item is expressed as $-W(t)*Q(t)*e^{-rt_w}$, simplified to $WCO*Q(t)$.

Bringing together the various components of the model results in the integration of the pieces. The certainty model in its final form is:

$$\begin{aligned}
 NPV = & \int_0^T \{ [P(t) * [q(t) * (1-d) * e^{-rt_d} + (1-q(t)) * (1-b(t)) * e^{-rt_c}] \\
 & - C(t) * [p(t) * (1-c) * e^{-rt_p} + (1-p(t)) * e^{-rt_f}] - W(t) * e^{-rt_w} - H(t) * e^{-rt_s}] * Q(t) \quad (1) \\
 & + j * (CA(t) - CM) - FC(\tau) \} * e^{-rt} * dt - FC - CM.
 \end{aligned}$$

Simplified to

$$\begin{aligned}
 NPV = & \int_0^T \{ [(dCI) + (ACI) - (cCO) - (ACO) - (WCO) - \\
 & (HCO)] * Q(t) + MSCI - FC(\tau) \} * e^{-rt} * dt - FC - CM. \quad (1a)
 \end{aligned}$$

The decision criterion is to accept the short and long-run financial management policy recommendations if the $NPV > 0$. More detailed discussion on this issue is presented in the next section.

D. Uncertainty Model

Because all of the variables are dependent on the level and trend of sales, the primary source of uncertainty in this model comes from the sales level.³ In contrast to the certainty model, the actual sales level of the firm can deviate from the forecasted sales level in this model. Furthermore, the forecasted sales level is the foundation of the production decision. The focus of the following analysis is the effect of forecasting errors in sales level on SRFM components and the impact on the value of the firm. It is assumed that the discount rate will be an appropriately determined risk adjusted rate of return.

1. Cash, Marketable Securities and Short-Term Borrowing

For cash and marketable securities, the major difference from the certainty model is that the net cash level may fall below the target cash level as a result of an unexpected decline in actual sales. When it happens, the firm will offset the shortfall by short term borrowing. The target cash level is a function of total forecasted sales level, i.e., $CM = f(Q)$, where $Q = \int_0^T Q_f(s) * ds$. In addition, the net cash level is now defined as

$$CA(t) = \int_0^t [CIF(s) - COF(s) - STB(s)] * ds,$$

where $STB(s)$ is the repayment of outstanding short-term loans in period s . The cash flow items associated with this working capital element are expressed as

$$-CM + j * \text{MAX}(0, CA(t) - CM) - k * \text{MAX}(0, CM - CA(t)),$$

or in simplified form:

$$-CM + \text{MSCI} - \text{STBCO}$$

2. Collection of Accounts Receivable

The items and relationships associated with accounts receivable are identical to those in the certainty model except that $Q(t)$ is replaced by $Q_a(t)$, the actual sales level. The associated cash inflow items are expressed as

$$P(t) * Q_a(t) * [(1-d) * q(t) * e^{-rt_d} + (1-q(t)) * (1-b(t)) * e^{-rt_c}].$$

or in simplified form:

$$\sim \quad \sim$$

$$(\text{dCI} + \text{ACI})$$

3. Inventory

In an uncertain world, it is reasonable for a firm to maintain a determined level of finished goods inventory in order to reduce the opportunity losses from not satisfying customer demand and the additional costs associated with fluctuating and overtime production. In addition to holding inventories in the forms of raw materials and goods-in-process as in the certainty model, the planned finished goods inventory, $n(t) \cdot Q_f(t)$, or PFGO, is introduced as a cushion to offset the uncertainty in the sales level. Its level depends on forecasted sales level (Q_f), production costs, holding costs and financing costs, as well as the production period and the trade discount, i.e.,

$$n(t) = f(Q_f(t), C(t), C'(t), W(t), W'(t), H(t), H'(t), k, t_s, c)$$

where

$$\frac{\partial n(t)}{\partial Q_f(t)}, \frac{\partial n(t)}{\partial C(t)}, \frac{\partial n(t)}{\partial W(t)}, \frac{\partial n(t)}{\partial H(t)}, \frac{\partial n(t)}{\partial H'(t)}, \frac{\partial n(t)}{\partial t_s}, \frac{\partial n(t)}{\partial k} < 0;$$

$$\frac{\partial n(t)}{\partial C'(t)}, \frac{\partial n(t)}{\partial W'(t)}, \frac{\partial n(t)}{\partial c} > 0.$$

In terms of cash flows, the total holding costs of planned inventory are expressed as $-H(t) \cdot (1+n(t)) \cdot Q_f(t) \cdot e^{-rt_s}$, or in simplified form (HCO*PFGO).

4. Payment of Accounts Payable

For accounts payable the only difference between the certain and uncertainty models is that the quantity of payments is related to the forecasted sales level and is adjusted for the planned finished goods inventory. The associated cash outflows are expressed as

$$-C(t) * Q_f(t) * (1+n(t)) * [(1-c) * p(t) * e^{-rt} + (1-p(t)) * e^{-rt_f}],$$

or
$$-(\tilde{c}CO + \tilde{A}CO * PFGO).$$

5. Other Cash Outflows

Other cash outflows are similar to the case for accounts payable. The difference in the other cash flows in relation to the certainty model is that the quantity of the other variable costs is related to the forecasted sales level and is adjusted for planned inventory level. Fixed costs will be the same as in the certainty model. The associated cash outflows are expressed as

$$-W(t) * Q_f(t) * (1+n(t)) * e^{-rt_w} - FC - FC(\tau) * e^{-rt}, \text{ or simplified to}$$

$$WCO * PFGO - FC - FC(\tau) * e^{-rt}$$

6. Forecasting Errors and Short Term Borrowing: The Uncertainty Dimension

Forecasting errors occur because actual sales are either greater or less than forecasted. However, in a planning model it is impossible to know ex post (actual) sales. Therefore, for modeling purposes we assume Q_a is an ex ante expression of actual sales and that it is a close proxy for ex post sales. The impact of forecasting error results is either insufficient or excess inventories. It is assumed

that the costs associated with forecasting errors are financed first by any net cash level in excess of the target cash balance, and then, if not fully covered, short term borrowing is used.

When the Q_a level is greater than the forecasted level, Q_f , the firm first tries to meet the additional demand with planned finished goods inventory. The next step uses any unplanned finished goods inventory accumulated through time, $UIA(t)$, defined as $\int_0^t [Q_f(s)*(1+n(s)) - Q_a(s)] * ds$ and, finally, the use of immediate production, $Q_a(t) - Q_f(t)*(1+n(t)) - UIA(t)$. It is assumed that the firm is able to achieve the immediate production by paying higher costs for raw materials and overtime labor, i.e., $C'(t)$ and $W'(t)$, where $C'(t) \gg C(t)$ and $W'(t) \gg W(t)$. These short term loans are assumed to be repaid as soon as the cash flows in. This component of forecasting error related cash outflow is expressed as

$$- \{e^{k(t_b(t)-t_s)} * [C'(t) + W'(t)] * \text{MAX}(0, Q_a(t) - Q_f(t)*(1+n(t)) - UIA(t)) * e^{-rt_b(t)},$$

or reduced to (SFECO).

When the $Q_f > Q_a$, the additional amount of finished products in that period will be added to the unintended inventory accumulation account and gives rise to additional inventory holding costs. It is assumed that this inventory will be held at a higher cost, $H'(t)$, until Q_a level exceeds the forecasted level to absorb the excessive unplanned inventories, and the bank loan is repaid with the positive net cash flows. The component of cash outflow associated with this forecasting error costs is expressed as

$$- \{e^{k(t_b(t)-t_s)} * H'(t) * \text{MAX}(0, Q_f(t) * (1+n(t)) + \text{UIA}(t) - Q_a(t))\} * e^{-rt_b(t)},$$

or simplified to (FSECO).

In summary, the uncertainty model is in the following form

$$\begin{aligned} \text{NPV} = & \int_0^T \{ \{ P(t) * Q_a(t) * [(1-d) * q(t) * e^{-rt_d} + (1-q(t)) * (1-b(t)) * e^{-rt_c}] \\ & - Q_f(t) * (1+n(t)) * \{ C(t) * [(1-c) * p(t) * e^{-rt_p} + (1-p(t)) * e^{-rt_f}] \\ & + W(t) * e^{-rt_w} + H(t) * e^{-rt_s} \} - k * \text{MAX}(0, \text{CM} - \text{CA}(t)) + j * \text{MAX}(0, \text{CA}(t) - \text{CM}) \} \\ & - \{ e^{k(t_b(t)-t_s)} * \{ [C'(t) + W'(t)] * \text{MAX}(0, Q_a(t) - Q_f(t) * (1+n(t)) - \text{UIA}(t)) \\ & + H'(t) * \text{MAX}(0, Q_f(t) * (1+n(t)) + \text{UIA}(t) - Q_a(t)) \} \} * e^{-rt_b(t)} - \text{FC}(\tau) \} * e^{-rt} * dt \\ & - \text{FC} - \text{CM}. \end{aligned} \quad (2)$$

In simplified form

$$\begin{aligned} \text{NPV} = & \int_0^T \{ [(\tilde{d}\text{CI} + \tilde{\text{ACI}}) - (\tilde{c}\text{CO} + \tilde{\text{ACO}} * \text{PFGO}) - (\text{WCO} * \text{PFGO}) - (\text{HCO} * \text{PFGO}) - \text{STBCO} \\ & \text{MSCI}] - [\text{SFECO} + \text{FSECO}] - \text{FC}(\tau) \} * e^{-rt} * dt - \text{FC} - \text{CM}. \end{aligned}$$

The decision criterion is to accept the short and long-run financial management policy recommendations if the $\text{NPV} > 0$. A more generalized model that assumes a multi-divisional firm with more than one product is presented in Appendix II. The decision criterion are the same as for (2).

III. IMPLICATION

It is now meaningful to turn to the discussion of the results generated by the models and the implications to the decision making

process. The discussion emphasizes the effect that changes in short run financial policies will have on NPV. The uncertainty model is used in the analysis because it is more comprehensive and closer to the real world.

A. Change in Credit Policy--Collection Effect

A change in credit policy can be considered as a decision of choosing between two mutually exclusive alternatives, where one is to continue current policy and the other is to pursue a proposed change in policy. The firm should choose the policy that creates the higher NPV.

In a narrow sense, a change in the credit policy implies changes in the values of the credit policy elements, namely, the trade discount, the discount and the credit periods.⁴ Changes in credit policy can affect the payment behavior of customers. For example, an increase in the trade discount may cause slower paying customers to pay earlier, resulting in a shorter collection period. Additionally, the interrelationships among the SRFM components and the sales levels can magnify the effect of the change. In particular, $Q_a(t)$, $Q_f(t)$, $q(t)$, $b(t)$, $n(t)$, $CA(t)$, $UIA(t)$, $t_b(t)$, CM , FC and $FC(\tau)$ will vary directly or indirectly with the credit policy elements. For example, it is expected that $Q_a(t)$, $Q_f(t)$ and $q(t)$, the payment pattern effects, will increase directly with the credit terms because new customers come in as a result of a more favorable credit policy and the current customers may change their payment behavior in response to the new credit policy. $CA(t)$, FC , $FC(\tau)$ and CM will increase indirectly, while $n(t)$ will

decrease indirectly, with an increase in $Q_a(t)$ or $Q_f(t)$. But $UIA(t)$, $t_b(t)$ and $b(t)$ can vary in either direction with the credit terms, depending on the situation.

Consider the case where a firm wanted to improve the timing of its collections. The firm proposed to speed up the payment behavior of its customers by increasing its trade discount from d_1 to d_2 , and shortening its discount period and credit period from t_{d1} to t_{d2} and t_{c1} to t_{c2} , respectively ($d_1 < d_2$, $t_{d1} > t_{d2}$, $t_{c1} > t_{c2}$). We assume the increase in d has a greater impact on sales than the decrease in t_d and t_c , so that there is a positive net induced sales effect resulting from the change in credit policy. The induced sales effect is composed of a short term gain in sales from competitors that will not be long lasting and, a long run gain in the sales level from attracting new customers from close substitutes and nonuser groups because the effective price of the product is lower. Finally, we assume the firm had completed a pilot study and had determined the information to be used for each variable and the key interrelationships among the variables.

The credit policy variables that are to be changed are underscored in (3) and the variables that will be affected by the change in collection policy are identifiable with an \uparrow in (3). According to the framework of the model, the NPV of the current credit policy is expressed as

$$\begin{aligned}
 NPV_1 = & \int_0^T \{ [P(t) * Q_{a1}(t)] * [(1-d_1) * q_1(t) * e^{-rt_{d1}} + (1-q_1(t)) * (1-b_1(t)) * e^{-rt_{c1}}] \\
 & - Q_{f1}(t) * (1+n_1(t)) * \{ C(t) * [(1-c) * p(t) * e^{-rt_p} + (1-p(t)) * e^{-rt_f}] \\
 & + W(t) * e^{-rt_w} + H(t) * e^{-rt_s} \} - k * \text{MAX}(0, CM_1 - CA_1(t)) + j * \text{MAX}(0, CA_1(t) - CM_1) \} \quad (3) \\
 & - \{ e^{k(t_{b1}(t) - t_s)} * [C'(t) + W'(t)] * \text{MAX}(0, Q_{a1}(t) - Q_{f1}(t) * (1+n_1(t)) - UIA_1(t)) \\
 & + H'(t) * \text{MAX}(0, Q_{f1}(t) * (1+n_1(t)) + UIA_1(t) - Q_{a1}(t)) \} * e^{-rt_{b1}(t) - FC_1(\tau)} * e^{-rt} * dt \\
 & - FC_1 - CM_1.
 \end{aligned}$$

The markings used in (3) are utilized in (3A) to identify the credit policy variables being changed and the variables sensitive to the change. Given the proposed change in credit policy, the NPV of the firm is expressed as

$$\begin{aligned}
 NPV2 = & \int_0^T \{ \{ P(t) * Q_{a2}(t) * [(1-d_2) * q_2(t) * e^{-rt} d_2 + (1-q_2(t)) * (1-b_2(t)) * e^{-rt} c_2] \\
 & - Q_{f2}(t) * (1+n_2(t)) * \{ C(t) * [(1-c) * p(t) * e^{-rt} p + (1-p(t)) * e^{-rt} f] \\
 & + W(t) * e^{-rt} w + H(t) * e^{-rt} s \} - k * \text{MAX}(0, CM_2 - CA_2(t)) + j * \text{MAX}(0, CA_2(t) - CM_2) \} \quad (3a) \\
 & - \{ e^{k(t_{b2}(t) - t_s)} * \{ [C'(t) + W'(t)] * \text{MAX}(0, Q_{a2}(t) - Q_{f2}(t) * (1+n_2(t)) - UIA_2(t)) \\
 & + H'(t) * \text{MAX}(0, Q_{f2}(t) * (1+n_2(t)) + UIA_2(t) - Q_{a2}(t)) \} * e^{-rt_{b2}(t) - FC_2(\tau)} * e^{-rt} * dt \\
 & - FC_2 - CM_2.
 \end{aligned}$$

The decision rule is that the proposed credit policy change should be adopted only if the value of the firm is increased, i.e., $NPV2 - NPV1 > 0$. The result is a positive collection effect. A comparable example could be developed that would illustrate a distribution effect related to a change in payment policy terms to suppliers.

B. Other Implications

The same line of considerations and decision criteria can be applied to a change in demand or other policy changes such as inventory, cash management and combinations of short run financial management policies. As more policy changes are involved, the interacting relationships among variables become more complicated and more insightful ideas can be generated from the model.

IV. CONCLUDING REMARKS

A primary contribution on the paper is to show that SRFM variables affect the magnitude and timing of cash flows and thereby are directly related to the value creation process. Without considering the interrelationships among SRFM elements through their relationships with sales and production levels, one might overstate the NPV of a firm and mistakenly change SRFM policies that would reduce the value of the firm. The mistake is more serious under uncertainty because there are costs associated with the forecasting error, which are closely related to the SRFM variables. By integrating the short-run financial management variables into the long-run financial planning process, fresh insights concerning the creation of a firm's value are introduced.

FOOTNOTES

¹Using a Markov process approach to study the payment behavior of retail department store customers, Kallberg and Saunders [15] found that $q(t)$ and $b(t)$, bad debt-loss, vary over time. Our research indicates that payment behavior varies among product lines and the distributions tend to be right skewed. Also we observed there are customers that pay before the discount period ends as well as customer paying after the credit period ends. Although we have not completed the study, it appears that payment behavior of manufacturing firms is different from retail customers.

²The bad debt loss, $b(t)$, refers to the portion of the accounts receivable which cannot be recovered at the end of the credit period when the firm writes off the uncollected items to the credit collecting agencies. Its level is a function of the payment pattern of the customer, the credit policy variables and the price and sales level, i.e., $b(t) = f(P(t), Q(t), d, t_d, t_c, q(t))$. The payment pattern variable, $q(t)$, is a function of the price and sales levels, as well as the credit policy variables, i.e., $q(t) = f(P(t), Q(t), d, t_d, t_c)$ where

$$\frac{\partial q(t)}{\partial P(t)}, \frac{\partial q(t)}{\partial Q(t)} \geq 0; \frac{\partial q(t)}{\partial d} > 0; \frac{\partial q(t)}{\partial t_d}, \frac{\partial q(t)}{\partial t_c} < 0$$

³Sartoris & Hill assumed the major sources of uncertainty came from the payment patterns of customers and the sales level. They discussed alternatives to solve the former source of uncertainty, but they did not develop an uncertainty model. In this section, we extend the S&H study by focusing on the uncertainty related to the sales level.

⁴The control of the quality of customers and the intensity of collection activities cannot be expressed even though they may have a similar impact on the SRFM variables and the subsequent value of the firm.

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APPENDIX I

Notations

$P(t)$: product price

$Q(t)$: sales level (in the certainty model)

$Q_a(t)$: an ex ante proxy for actual sales level (in the uncertainty model)

$Q_f(t)$: forecasted sales level (in the uncertainty model)

$C(t)$, $C'(t)$: raw material costs per unit

$W(t)$, $W'(t)$: other variable costs per unit

$H(t)$, $H'(t)$: average inventory (raw material and work-in-process) holding and ordering cost per unit

FC: fixed costs

FC(t): periodic fixed costs

CM: target cash balance

UIA(t): unplanned finished goods inventory accumulation level

CA(t): net cash level

T: strategic planning horizon

d: trade discount to the customers

$q(t)$: fraction of sales collected at the end of the discount period; can be extended to include multiple collection patterns

$b(t)$: fraction of sales uncollected (bad debt loss)

c: trade discount offered by the suppliers

$p(t)$: fraction of raw material costs paid at the end of the discount period (can be extended to include multiple payment periods)

$n(t)$: planned finished goods inventory as a percentage of $Q_f(t)$

j: short term lending rate (return on marketable securities)

k: short term borrowing rate

- r : discount rate (risk-free rate in the certainty model;
cost of capital in the uncertainty model)
- t_c : credit period to the customers
- t_d : discount period to the customers ($t_d < t_c$)
- t_f : credit period from the suppliers
- t_p : discount period from the suppliers ($t_p < t_f$)
- t_w : deferral period of the other variable costs
- t_s : production period
- $t_b(t)$: repayment period of short term borrowing

The exogenous variables are $P(t)$, $C(t)$, $C'(t)$, $W(t)$, $W'(t)$, $H(t)$, $H'(t)$, T , j , k , r , t_w and t_s . The control (policy) variables are d , t_d , t_c , c , t_p and t_f . The remainder are endogenous variables, i.e., $Q(t)$, $Q_a(t)$, $Q_f(t)$, CM , $CA(t)$, FC , $FC(t)$, $UIA(t)$, $q(t)$, $b(t)$, $p(t)$, $n(t)$ and $t_b(t)$.

APPENDIX II

For simplicity, we have an implicit assumption about the firm in the development of our model, namely, we are assuming a single product/division firm. In practice, we rarely come across firms with this nature. In the real world, most of the firms are multi-divisional with more than one product line and competing in several industries. In order to capture this real world phenomenon in our model and to make it more applicable, a generalized version of the model is presented in this appendix.

Assume that we have an m divisions/product lines firm which is competing in m industries. In addition, the assumptions used in developing our model are applicable here. For each division, it has its own credit policy for its customers and the payment policy to its suppliers. Besides, different price levels, forecasted and actual sales levels, inventory policies, wage rates, unit raw material costs and product periods are set up for different divisions/product lines. This set of independency assumption is reasonable given that all divisions are competing in different industries. So each division follows the rules of its own industry which are likely to vary across industries.

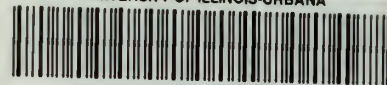
On the other hand, there are several common variables for all divisions since they are part of the same firm. For instance, they share the same cost of capital, the same rate of short term lending and borrowing, the same warehouse facilities and so the unit holding cost for inventory, the same fixed outlays, the same target cash balance and the same borrowing period. For each period, the negative

net cash levels of the deficit units are offset by the positive net cash levels of the surplus divisions before the firm goes for a short-term bank loan. This kind of treatment may not be acceptable from the standpoint of divisional performance evaluation since it undervalues the performance of surplus units but overvalues that of deficit units. But, we should be reminded that this study focuses on the profitability and thus the value of the firm as a whole.

Formally, this generalized version can be expressed as

$$\begin{aligned}
 NPV = & \int_0^T \left\{ \sum_{\ell=1}^m \left\{ P_{\ell}(t) * Q_{a\ell}(t) * [(1-d_{\ell}) * q_{\ell}(t) * e^{-rt_{d\ell}} + (1-q_{\ell}(t)) * (1-b_{\ell}(t)) * e^{-rt_{c\ell}}] \right. \right. \\
 & - Q_{f\ell}(t) * (1+n_{\ell}(t)) * \{ C_{\ell}(t) * [(1-c_{\ell}) * p_{\ell}(t) * e^{-rt_{p\ell}} + (1-p_{\ell}(t)) * e^{-rt_{f\ell}}] \\
 & + W_{\ell}(t) * e^{-rt_{w\ell}} + H(t) * e^{-rt_{s\ell}} \} \} - k * \text{MAX}(0, CM - \sum_{\ell=1}^m CA_{\ell}(t)) \\
 & + j * \text{MAX}(0, \sum_{\ell=1}^m CA_{\ell}(t) - CM) - \sum_{\ell=1}^m \{ e^{k(t_b(t) - t_{s\ell})} * [C'_{\ell}(t) + W'_{\ell}(t)] \\
 & * \text{MAX}(0, Q_{a\ell}(t) - Q_{f\ell}(t) * (1+n_{\ell}(t)) - UIA_{\ell}(t)) + H'(t) * \text{MAX}(0, Q_{f\ell}(t) * (1+n_{\ell}(t)) \\
 & + UIA_{\ell}(t) - Q_{a\ell}(t)) * e^{-rt_{b(t)}} \} - FC(\tau) \} * e^{-rt} * dt - FC - CM.
 \end{aligned}$$

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